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THE PHYSICAL PROPAGATOR OF A SLOWLY MOVING CHARGE

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Abstract We consider an electron which is electromagnetically dressed in such a way that it is both gauge invariant and that it has the associated electric and magnetic fields expected of a moving charge. We study the propagator of this dressed electron and, for small velocities, show explicitly at one loop that at the natural (on-shell) renormalisation point, $p_0 = m, \boldsymbol{p} = m\boldsymbol{v}$, one can renormalise the propagator multiplicatively. Furthermore the renormalisation constants are infrared finite. This shows that the dressing we use corresponds to a slowly moving, physical asymptotic field.

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It is well known that there are difficulties in finding the correct asymptotic fields in theories with massless particles and/or confinement. These problems plague the gauge theories of the electromagnetic and strong interactions. They find expression in infra-red divergences in the perturbative expansion of Green's functions with external lines corresponding to charged particles. The standard approaches to the infra-red problem dress the charged particles by adding an (infinite) number of the massless gauge bosons. This means that descriptions of charged particles should be based around coherent states and not on a Fock space description. (For a concise review of the infra-red problem and a critique of standard responses see Supplement 4 of Ref. 1.) We also recall here that it is known that any description of a state corresponding to a charged particle must be both non-local[2−5] and non-covariant[6,7,5]. Physically this merely corresponds to the need to dress an electron, say, with an electromagnetic cloud which essentially falls off away from the charge as $1/R$ and whose exact form depends upon the velocity of the charge at its centre. Although the need for such a dressing can often be sidestepped in the calculation of QED scattering processes, it is fundamental to any description of charged states in this theory. In QCD where charges are confined by their mutual interactions, one finds[5] that the dressings play an even greater role, and, e.g., they are required if we want to construct colour charges in a well-defined way.

There is a long history of trying to understand how dressings can be incorporated into gauge theories[8−13]. In a recent series of papers[14−17], which are summarised and extended in Ref. 5, two of us have proposed a new approach to gauge theories where the dressing associated with charged states is taken fully into account from the beginning. Two criteria lie at the heart of this approach: the dressed charged particle, since it describes a physical field, should be gauge invariant and secondly the associated electric and magnetic fields should be those we expect of a charged particle. Such a dressed field was, in fact, suggested many years ago by Dirac (see Sect. 80 of Ref. 18). He pointed out that the gauge invariant combination

$$\psi_{\mathrm{c}}(x) = \exp\left(ie\frac{\partial_i A_i}{\nabla^2}(x)\right)\psi(x)\,, \tag{1}$$

made out of a fermion and a non-local cloud of vector potentials could represent an electron since it generates a Coulomb electric field that we would like to associate with a physical electron. To see this, one may make use of the equal time commutator, $[E_i(\boldsymbol{x}), A_j(\boldsymbol{y})] = i\delta_{ij}\delta(\boldsymbol{x}-\boldsymbol{y})$, and see that for an eigenstate, $|\epsilon\rangle$ of the electric field with eigenvalue, $\epsilon_i(\boldsymbol{x})$, the introduction of such a dressed charge as (1) alters the electric field in the way we expect

of a physical static charge:

$$E_i(\boldsymbol{x})\psi_{\rm c}(\boldsymbol{y})|\epsilon\rangle = \left(\epsilon_i(\boldsymbol{x}) + \frac{e}{4\pi}\frac{\boldsymbol{x}_i - \boldsymbol{y}_i}{|\boldsymbol{x}-\boldsymbol{y}|^3}\right)\psi_{\rm c}(\boldsymbol{y})|\epsilon\rangle\,. \tag{2}$$

It is evident that the description (1) of an electron is, as one expects, non-local and non-covariant. It describes a static charge (one can easily see for example that (1) does not possess any associated magnetic field) and should only be used for this purpose. In a recent paper[5] a description of a dressed abelian charge, moving with a velocity $\boldsymbol{v}$, analogous to (1) was found. For a charge moving in the x^1-direction, so that $\boldsymbol{v} = (v, 0, 0)$ this is

$$\begin{aligned}\psi_{\boldsymbol{v}} =& \exp\Bigg(-\frac{e}{4\pi}\frac{1}{\sqrt{1-v^2}} \\ &\times \int d^3z \frac{(1-v^2)\partial_1 A_1(x^0,\boldsymbol{z}) + \partial_2 A_2(x^0,\boldsymbol{z}) + \partial_3 A_3(x^0,\boldsymbol{z}) - vE_1(x^0,\boldsymbol{z})}{\left(\dfrac{(x_1-z_1)^2}{1-v^2} + (x_2-z_2)^2 + (x_3-x_3)^2\right)^{\frac{1}{2}}}\Bigg)\psi(x)\,,\end{aligned} \tag{3}$$

which is easily seen to be gauge invariant and to reduce to (1) in the limit, $\boldsymbol{v} \to 0$. From the fundamental commutators one can straightforwardly find that the magnetic and electric fields associated with (3) are just those that we would expect from a charge moving with such a velocity $\boldsymbol{v}$.

One might wonder if this argument, based on the use of free commutators, really holds in the interacting theory. In this letter we will calculate the one loop propagator for such a non-static dressed charge and show that this interpretation of the dressed field indeed persists in the quantum theory. For computational simplicity we will restrict ourselves to small velocities and work at order $\boldsymbol{v}$. This means that we will find the one loop propagator of

$$\psi_{\boldsymbol{v}}(x) = \exp\left(ie\frac{\partial_j A_j + v_i E_i}{\nabla^2}\right)\psi(x)\,. \tag{4}$$

Work on the, much more involved, relativistic case is in progress. Note further that e^2 terms in (4) may be dropped since in the one loop propagator they will only yield tadpoles which can be dropped in BRST respecting regularisation schemes.

Before proceeding to the calculation of the propagator itself we should briefly recall what is known about the two-point Green's function of the Lagrangian fermion at one loop and also the equivalent propagator for the dressed, static charge, (1). The renormalisation of this Green's function of the Lagrangian field requires a mass renormalisation and a wave

function renormalisation. Working in a general covariant gauge, one finds that the mass renormalisation is gauge parameter independent, which indicates its physical significance (see also Ref. 19). The wave function renormalisation displays the infra-red singularities mentioned in the introduction if one tries to renormalise this propagator on shell. This shows that the Lagrangian fermion is not a good asymptotic field and the need for a dressing.

The one loop propagator of the dressed static fermion, (1), was carried out in Ref. 5. It was found that the propagator was gauge independent (the calculation was carried out both in a general Lorentz gauge and in Coulomb gauge) and that the mass renormalisation required was just the standard one for the usual fermion propagator. An on-shell wave function renormalisation was performed for the static mass shell scheme (i.e., $p_0 = m, \boldsymbol{p} = 0$) whose use is, as discussed above, a natural consequence of (2) and no difficulties were encountered. This was interpreted as strong evidence that (1) is a good asymptotic description of a static charge. It was further checked that (1) is *not* suitable for describing moving charges: any attempt at renormalising the propagator of (1) at a non-static mass shell was shown to founder upon non-multiplicative structures and infra-red divergences. This led to the conjecture in Ref. 5 that it would be possible to renormalise the propagator of (3) at the relevant non-static mass shell. After having set the scene we may now proceed to the calculation of the one loop propagator of (4) and verify the above conjecture.

There are two ways to calculate this propagator. Either one may work in what we call the *dressing gauge*, where the dressing vanishes[1] or one can use a general Lorentz gauge. We have followed both approaches and checked that the same results emerge. The dressing gauge here is $\partial_i A_i + v_i E_i = 0$ and one may readily see that the photon propagator in this gauge is

$$D_{\mu\nu}(k) = \frac{1}{k^2}\Big(- g_{\mu\nu} - \frac{(\boldsymbol{k}^2 - 2k_0 v\cdot k)k_\mu k_\nu}{\boldsymbol{k}^4} - \frac{k_0 - k\cdot v}{-\boldsymbol{k}^2}(k_\mu \eta_\nu + \eta_\mu k_\nu) - \frac{k_0}{-\boldsymbol{k}^2}(k_\mu v_\nu + v_\mu k_\nu)\Big) + O(\boldsymbol{v}^2)\,, \tag{5}$$

where we have introduced the temporal vector, $\eta = (1,0,0,0)$ and we point out that $v^\mu = (0, \boldsymbol{v})$ implies that $v\cdot k = -\boldsymbol{v}\cdot\boldsymbol{k}$. Working in this gauge one merely needs to calculate the diagram of Fig. 1.a. In general we have contributions to order e^2 from both interaction

[1] In general for three-point functions and higher there is no gauge where all the dressings would be expected to vanish. Note also that if we have more than one charged particle, there is a phase factor as well as the various individual dressings[5].

vertices and the expansion of the dressing in (4) and in an arbitrary Lorentz gauge all of the diagrams of Fig. 1 must be calculated. We stress that the gauge parameter dependence then cancels and that the result is the same as that found in our dressing gauge.

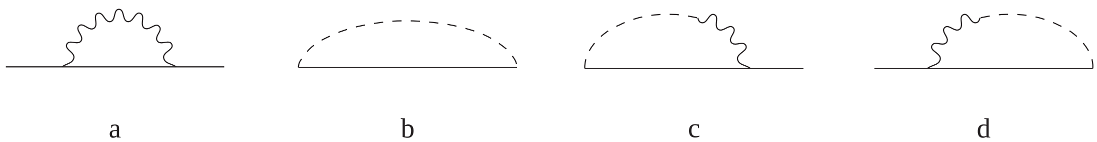

Figure 1 The diagrams which yield the one loop dressed propagator. In the appropriate dressing gauge only Fig. 1.a contributes. In a general gauge all the diagrams must be evaluated. The dashed lines indicate the projection of the photon propagator from the ($\boldsymbol{v}$-dependent) vertices in the dressing.

Since we work to order $\boldsymbol{v}$ our propagator is a sum of two terms: those which one has in Coulomb gauge and various new terms linear in the velocity. The former terms have been considered in Ref.'s 20 (where a list of integrals may be found) and 5. The latter terms yield some new ultraviolet divergences. Dropping, for the moment, the finite terms we find with a little effort the following UV divergences which are proportional to $\boldsymbol{v}$ in the self-energy:

$$-i\Sigma^{\mathrm{UV}}_{\boldsymbol{v}} = \frac{i\alpha}{4\pi}\frac{1}{2-\omega}\left(\frac{8}{3}p_0 \not{v} - \frac{8}{3}p\cdot v\not{\eta}\right) + \cdots\,, \tag{6}$$

and $\alpha = e^2/4\pi$. Since these tensor structures do not appear in the original Lagrangian we would seem not to be able to perform a multiplicative renormalisation. Although this difficulty is common in axial gauge studies, its appearance would be unattractive. Recall also that in Coulomb gauge it has been shown by Heckathorn[21] that the UV divergences in the propagator are multiplicatively renormalisable. We will see, however, that these divergences can in fact be dealt with straightforwardly. It should also perhaps be noted that this apparent difficulty is not present on-shell. If we use $\bar\psi\gamma_\mu\psi = \bar\psi\psi p_\mu/m$, which holds for on-shell spinors, then it immediately follows that the UV divergences of (6) vanish on-shell.

To renormalise the propagator through mass and wave-function counterterms in a general and systematic way consider now the following (matrix) multiplicative renormalisation scheme:

$$\psi^B_{\boldsymbol{v}} = \left(Z_2^{\frac12}\boldsymbol{I} + \frac{Z'}{Z_2^{\frac12}}\not{\eta}\not{v}\right)\psi_{\boldsymbol{v}}\,, \tag{7}$$

or equivalently at lowest order

$$\psi_{\boldsymbol{v}}^{B}=\sqrt{Z_2}\exp\{-i\frac{Z'}{Z_2}\sigma^{\mu\nu}\eta_\mu v_\nu\}\psi_{\boldsymbol{v}}\,, \tag{8}$$

together with the mass shift

$$m\to m-\delta m\,. \tag{9}$$

It is easy to see that in the self-energy we then have to order $\boldsymbol{v}$ the counterterm:

$$-i\Sigma_{\boldsymbol{v}}^{\rm count}=i\delta Z_2(\not p-m)+2iZ'(p_0\not v-p\cdot v\not\eta)+i\delta m\,, \tag{10}$$

with $Z_2=1+\delta Z_2$ and where we made use of $\not\eta\not v=-\not v\not\eta$. We should also recall that δZ_2 and Z' are both of order α. We see that a suitable choice of Z' cancels the UV divergences of (6). Schematically the full self-energy, $-i\Sigma_{\boldsymbol{v}}^{\rm tot}=-i\Sigma_{\boldsymbol{v}}-i\Sigma_{\boldsymbol{v}}^{\rm count}$, now has the form

$$-i\Sigma_{\boldsymbol{v}}^{\rm tot}\equiv m\alpha+\not p\,\beta+p_0\not\eta\,\delta+m\not v\,\epsilon\,, \tag{11}$$

where the functions α, β, δ and ϵ depend upon p^2, p_0 and $p\cdot v$.

The on-shell renormalisation condition is that at the physical mass, m, the dressed propagator, $iS_{\boldsymbol{v}}$, should have the form of a free field propagator, i.e., with a simple pole at m, the Lagrangian mass, and a residue of one at that pole. From (11) it then follows that

$$\tilde\alpha+\tilde\beta+\frac{p_0^2}{m^2}\tilde\delta+\frac{p\cdot v}{m}\tilde\epsilon\equiv 0\,, \tag{12}$$

where the tildes on the functions denote that they are evaluated at $p^2=m^2$. We stress that the non-covariant nature of these functions implies that this condition should hold for arbitrary p_0 and $p\cdot v$ as long as the particle is on-shell. We will return to this in a moment.

With the above notation for the self-energy we can write the propagator as

$$\begin{aligned}iS_{v}=i\frac{\not p+m}{p^2-m^2}-\frac{1}{p^2-m^2}\Big\{&(2m^2\tilde\Delta+\tilde\beta)\not p\\&+(2m^2\tilde\Delta+\tilde\alpha+2\tilde\beta)m-p_0\not\eta\,\tilde\delta-m\not v\,\tilde\epsilon\Big\}\,,\end{aligned} \tag{13}$$

where

$$\tilde\Delta(p_0,p\cdot v)=\Big(\frac{\partial\alpha}{\partial p^2}+\frac{\partial\beta}{\partial p^2}+\frac{p_0^2}{m^2}\frac{\partial\delta}{\partial p^2}+\frac{p\cdot v}{m}\frac{\partial\epsilon}{\partial p^2}\Big)\Big|_{p^2=m^2}\,. \tag{14}$$

We see that if the second term is to vanish we must require that the three-vector component of p is proportional to $\boldsymbol{v}$. Let us now take

$$p^\mu = (m,\, m\boldsymbol{v})\,, \tag{15}$$

which is what we could expect from our introductory considerations: $p^2 = m^2 + O(v^2)$ and the three-momentum is that of a particle with velocity $\boldsymbol{v}$. Our prediction is that this will be a good on-shell renormalisation point for this dressed propagator. It then follows that we require for the term in the curly brackets to vanish on-shell that

$$\bar\alpha + \bar\beta + \bar\delta = 0\,, \tag{16}$$

which is just (12) with the choice (15), and

$$\bar\delta = \bar\epsilon \tag{17}$$

where bars over the functions mean that they are now evaluated at $p^\mu = (m,\, m\boldsymbol{v})$, our physically motivated choice of on-shell renormalisation point. Finally we also have

$$2m^2\bar\Delta + \bar\beta - \bar\delta = 0\,. \tag{18}$$

We now want to show that these conditions are indeed satisfied by the explicit expressions for the propagator from Fig. 1 if the counterterms are chosen properly. Using the integrals of Ref. 20 one finds after some work the following expressions for α, β, δ and ϵ:

$$\begin{aligned}
\tilde\alpha =& i\frac{\delta m}{m} + \frac{i\alpha}{4\pi}\left[-\frac{4}{\hat\epsilon} - 10 - 2\frac{p_0}{\mathbf{p}}\chi\right] - i\delta Z_2 - \frac{i\alpha}{4\pi}\frac{2p\cdot v}{\mathbf{p}^3}\left[2p_0\mathbf{p} + m^2\chi\right]\,,\\
\tilde\beta =& i\frac{\alpha}{4\pi}\left[\frac{1}{\hat\epsilon} + 6 - \frac{4p_0^2}{\mathbf{p}^2} + \left(\frac{2p_0}{\mathbf{p}} - \frac{2p_0^3}{\mathbf{p}^3}\right)\chi\right]\\
&\quad + i\delta Z_2 + i\frac{\alpha}{4\pi}\frac{2p\cdot v}{\mathbf{p}^5}\left[6\mathbf{p}p_0^3 + 2\mathbf{p}^3p_0 + 3m^2(p_0^2+\mathbf{p}^2)\chi\right]\,,\\
\tilde\delta =& i\frac{\alpha}{4\pi}\left[-4 + \frac{4p_0^2}{\mathbf{p}^2} - \left(\frac{2p_0}{\mathbf{p}} - \frac{2p_0^3}{\mathbf{p}^3}\right)\chi\right] - 2iZ'\frac{p\cdot v}{p_0}\\
&- i\frac{\alpha}{4\pi}\frac{2p\cdot v}{9p_0\mathbf{p}^5}\left[\frac{12}{\hat\epsilon}\mathbf{p}^5 + 58\mathbf{p}^5 - 24\mathbf{p}^3p_0^2 + 18\mathbf{p}p_0^4 + (9p_0^5 - 15\mathbf{p}^2p_0^3 + 18\mathbf{p}^4p_0)\chi\right]\,,\\
\tilde\epsilon =& 2iZ'\frac{p_0}{m} + i\frac{\alpha}{4\pi}\frac{2p_0}{9m\mathbf{p}^3}\left[\frac{12}{\hat\epsilon}\mathbf{p}^3 + 46\mathbf{p}^3 + 6\mathbf{p}p_0^2 + (3p_0^3 + 9\mathbf{p}^2p_0)\chi\right]\,,
\end{aligned} \tag{19}$$

where

$$\mathbf{p} = |\boldsymbol{p}|\,,\quad \chi = \ln\left(\frac{p_0 - \mathbf{p}}{p_0 + \mathbf{p}}\right)\,,\quad \text{and}\quad \frac{1}{\hat\epsilon} = \frac{1}{2-\omega} - \gamma_E + \ln 4\pi\,. \tag{20}$$

In the limit $\boldsymbol{v} \to 0$ we so obtain the results for the dressed static propagator (note that there are no $1/\mathbf{p}$ singularities).

One sees that the condition (12) is satisfied for arbitrary values of p_0 and $p \cdot v$ only if

$$\delta m = \frac{\alpha}{4\pi}(\frac{3}{\hat{\epsilon}} + 4)m\,. \tag{21}$$

This may be recognised as exactly the standard mass shift for the fermion propagator which is known[22,23,19] to be gauge parameter independent in the class of Lorentz gauges and was also found for the static, i.e., $\boldsymbol{v} \to 0$, dressed propagator[5].

We now have at the physical mass shell

$$\begin{aligned}
\bar{\alpha} =& i\frac{\delta m}{m} + i\frac{\alpha}{4\pi}\left[-\frac{4}{\hat{\epsilon}} - 6\right] - i\delta Z_2\,,\\
\bar{\beta} =& i\frac{\alpha}{4\pi}\left[\frac{1}{\hat{\epsilon}} + \frac{10}{3}\right] + i\delta Z_2\,,\\
\bar{\delta} =& i\frac{\alpha}{4\pi}\left[-\frac{4}{3}\right]\,,\\
\bar{\epsilon} =& i\frac{\alpha}{4\pi}\left[\frac{8}{3\hat{\epsilon}} + \frac{52}{9}\right] + 2iZ'\,,
\end{aligned} \tag{22}$$

which means that (16) is automatically satisfied — as we would expect because of its close relation to (12). Eq. 17 now requires that

$$Z' = -\frac{\alpha}{4\pi}\left(\frac{4}{3\hat{\epsilon}} + \frac{32}{9}\right)\,. \tag{23}$$

To obtain $\bar{\Delta}$ at order $\boldsymbol{v}$ we just need $\overline{\partial\alpha/\partial p^2}$ and similar for β and δ (the analogous derivative of ϵ does not contribute at this order in $\boldsymbol{v}$. One finds

$$\begin{aligned}
\overline{\frac{\partial\alpha}{\partial p^2}} =& \frac{i\alpha}{4\pi}\frac{1}{2m^2}\left[2\ln\frac{\lambda^2}{m^2} + \frac{4}{3}\right]\,,\\
\overline{\frac{\partial\beta}{\partial p^2}} =& \frac{i\alpha}{4\pi}\frac{1}{2m^2}\left[-\frac{2}{3}\ln\frac{\lambda^2}{m^2} - \frac{122}{45}\right]\,,\\
\overline{\frac{\partial\delta}{\partial p^2}} =& \frac{i\alpha}{4\pi}\frac{1}{2m^2}\left[-\frac{4}{3}\ln\frac{\lambda^2}{m^2} - \frac{148}{45}\right]\,.
\end{aligned} \tag{24}$$

where we have introduced a photon mass λ to regulate the infra-red sector. With these one has that the various infra-red divergences cancel and

$$\bar{\Delta} = \frac{i\alpha}{4\pi}\frac{1}{2m^2}\left[-\frac{14}{3}\right]\,. \tag{25}$$

This means that the condition (18) can be reexpressed as

$$Z_2 = 1 - \frac{\alpha}{4\pi}\frac{1}{\hat{\epsilon}}\,. \qquad (26)$$

This we recognise as the static wave function renormalisation constant. This concludes the renormalisation of the propagator to first order in the velocity. A finite result has been obtained and, up to the need for the matrix rotation parameterised by Z', we have been able to use completely standard techniques. This fully supports the conjecture in Ref. 5.

Another consequence of the results of this paper is that we have introduced a class of gauges parameterised by the vector, $\boldsymbol{v}$, which are infra-red finite. As well as these gauges we are only aware of the Yennie[24,25] gauge and the Coulomb gauge as yielding infra-red finite propagators. It should be considered that our propagator depends upon two vectors, η and $\boldsymbol{v}$, and we recall the many difficulties with axial gauge calculations[26].

The next extensions of this calculation to other QED Green's functions are rather clear: studies for relativistic velocities together with dressed vertices and higher Green's functions need to be considered. We hope that explicit results will further clarify the principles underlying the infra-red finiteness of these calculations.

The non-abelian extension of this work is still lacking, although the one-loop dressed quark propagator, whether static or not, is just its QED equivalent modulo a colour factor at this order in the coupling. Higher loop studies need to be performed. Asymptotic freedom means that at short distances perturbative dressings, like those of QED, are sufficient. At larger distances and higher orders in the coupling differences appear and the gluonic dressing around a quark becomes more and more complicated[5,15]. Non-perturbatively it is impossible to dress the quarks[15,5] and asymptotic quark states cannot be constructed. This does not mean that the perturbatively dressed fields do not have a physical significance and we would for example suggest that their use could be of interest in the heavy quark effective theory.

In summary we have studied the one loop propagator of a fermion dressed in such a way as to correspond to a charge moving with a small velocity $\boldsymbol{v}$. It was shown that the renormalisation procedure singled out the expected renormalisation point, $p = (m, m\boldsymbol{v})$ in accord with previous conjectures[5]. We interpret this as strong evidence that the dressed fermion (3) is a good asymptotic field. The renormalisation of the propagator at higher

orders in the velocity and indeed a general verification of our conjectures for the propagator of the dressed field (3) will be presented elsewhere.

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